IMPROVED METHODOLOGY OF SHIP HYDROELASTIC ANALYSIS

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ABSTRACT

The paper presents improved methodology of ship hydroelasticity analysis, based on the modal superposition method and including structural, hydrostatic and hydrodynamic models. Structural beam model is used comprising a reliable advanced thin-walled girder theory taking into account shear influence on torsion as well as the contribution of bulkheads and the engine room structure, as structure discontinuities, to the ship hull stiffness. Consistent restoring stiffness is included in the analysis via hydrostatic model. Hydrodynamic loading and added mass and hydrodynamic damping are a part of the hydrodynamic model and are determined based on the radiation / diffraction theory. Also, the analysis of springing effect on the ship fatigue life is introduced using the combination of the improved hydroelastic model and 3D FEM substructure model. It is shown that the improved beam hydroelastic model can be efficiently used in the assessment of stress concentrations of ship structural details. Applicability of the developed theory is proved within global hydroelastic analysis of a 11400 TEU container ship as well as stress concentration determination in the selected structural detail.

KEYWORDS

Hydroelasticity; Improved beam model; Hydrodynamics; Restoring stiffness; Springing; Stress concentrations

1. INTRODUCTION

It is well known that the hydroelastic problem can be solved at different levels of complexity and accuracy[1,2]. In this paper the sophisticated beam model is coupled with 3D hydrodynamic model in order to investigate the theoretical and practical thresholds of beam model applicability. Due to that the existing hydroelastic model is extended to determination of stress concentration transfer functions which are necessary for assessment of fatigue damage/life of ship structural details. Stress concentration transfer function calculation represents the first step in the fatigue damage assessment procedure[3]. The global hydroelastic response is calculated in the frequency domain using beam structural model and the modal displacements are imposed to the 3D FEM fine mesh substructure model. Stress concentration RAOs are then obtained for the considered structural detail. The global and local hydroelastic responses are finally verified through the comparison with those obtained by fully coupled 3D FEM + 3D BEM hydroelastic model[4].

2. MATHEMATICAL MODEL

2.1 Structural model

The advanced beam model takes shear influence on bending and torsion into account in a reliable way, as well as the contribution of transverse bulkheads and engine room structure to the ship hull global stiffness, so it can give quite accurate results.

Total beam deflection and twist angle consist of pure bending and torsion, respectively, and their shear contribution[5]

$$w = w_b + w_s = w_b - \frac{EI_b}{GA_s}\frac{\mathrm{d}^2 w_b}{\mathrm{d}x^2}, \quad \psi = \psi_t + \psi_s = \psi_t - \frac{EI_w}{GI_s}\frac{\mathrm{d}^2\psi_t}{\mathrm{d}x^2}, \tag{1}$$

where I_b is moment of inertia of cross-section, A_s is shear area, I_w is warping modulus and I_s is shear inertia modulus. E and G are Young's and shear modulus, respectively. One can see that there is an analogy between bending and torsion[5,6,7]

$$A_s = \frac{Q^2}{\int\limits_A \tau_Q^2 \mathrm{d}A}, \quad I_s = \frac{T_w^2}{\int\limits_A \tau_w^2 \mathrm{d}A}, \tag{2}$$

where Q and T_w are shear force and torque due to restrained warping, and τ_Q and τ_w are corresponding shear stresses, respectively.

The effect of large number of transverse watertight and support bulkheads taken into account as[7]

$$I_t^* = \left[1 + \frac{a}{l_1} + \frac{4(1+\nu)C}{I_t l_0}\right] I_t, \qquad C = \frac{U}{E\psi_t'^2}, \tag{3}$$

where, according to Figure 1, a is the web height of bulkhead girders, l_0 is the bulkhead spacing, $l_1 = l_0 - a$ is the net length, C is the energy coefficient, ν represents Poisson's ratio, and U is the bulkhead grillage and stool strain energy due to warping of cross-section. Warping shape function can be assumed by the following expression:

$$u(y,z) = \bar{u}(y,z)\psi_t' = -y\left\{(z-d) + \left[1 - \left(\frac{y}{b}\right)^2\right]\frac{z^2}{H}\left(2 - \frac{z}{H}\right)\right\}\psi_t', \tag{4}$$

where H is the ship height, b is one half of bulkhead breadth, d is the distance of warping centre from double bottom centroid, while y and z are transverse and vertical coordinates, respectively.

The bulkhead grillage strain energy is defined as[8]

$$U_g = \frac{1}{1-\nu^2}\left[\frac{116H^3}{35b}i_y + \frac{32b^3}{105H}i_z + \frac{8Hb}{75}\nu\left(i_y + i_z\right) + \frac{143Hb}{75}(1-\nu)i_t\right]E\psi'^2 \tag{5}$$

where i_y, i_z and i_t are the average moments of inertia of cross-section and torsional modulus per unit breadth, respectively. The stool strain energy is given as

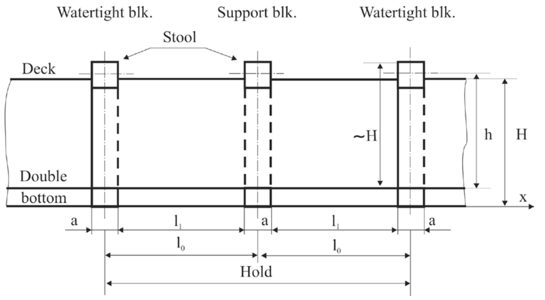

Figure 1: Longitudinal section of container ship hold

$$U_s = \left[\frac{12h^2 I_{sb}}{b} + 72(1+\nu)\frac{h^2}{b^3}\frac{I_{sb}^2}{A_s} + \frac{9bI_{st}}{10(1+\nu)} \right] E\psi'^2 , \tag{6}$$

where I_{sb}, A_s and I_{st} are the moment of inertia of cross-section, shear area and torsional modulus, respectively. Quantity h is the stool distance from the inner bottom.

Ultra Large Container Ships have relatively short engine room structure with length of about a half of ship breadth. Such structure behaves like a segment of the open cross section with increased torsional stiffness due to decks contribution[9]. The effective torsional modulus which includes both open cross-section and deck segments can be written in the form:

$$\tilde{I}_t = (1+C) I_t^{\circ}, \tag{7}$$

where I_t° is the torsional stiffness of an open cross-section segment and C is energy coefficient, defined as

$$C = \frac{\sum E_i}{E_t} = \frac{4(1+\nu)t_1\left(\frac{a}{b}\right)^3 \left(|w_D| + |w_B|\right)^2 k}{\left[1 + 2(1+\nu)\left(\frac{a}{b}\right)^2\right] I_t^{\circ} a}, \qquad k = \sum \frac{V_i}{V_1}\left(\frac{h_i}{h_1}\right)^2 , \tag{8}$$

where V_i represents volume of particular deck, w_D and w_B are values of deck and bottom warping functions, respectively. The other geometric quantities in the above formula are illustrated in Figure 2.

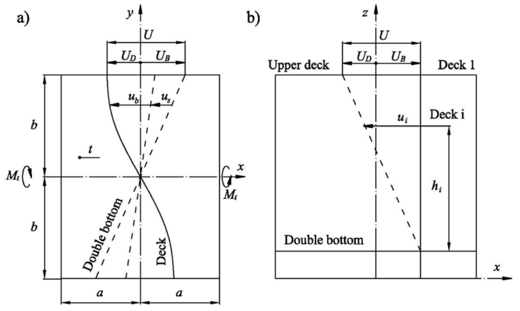

Figure 2: Engine room deck deformation and double bottom rotation, a) bird's eye, b) lateral view

Finally all stiffness contributions are summed up in a well-known dry natural vibrations matrix equation

$$\left(\mathbf{K}-\Omega^2\mathbf{M}\right)\boldsymbol{\delta}=\mathbf{0}, \tag{9}$$

where $\mathbf{K}$ is stiffness matrix, $\mathbf{M}$ is mass matrix, Ω is dry natural frequency and $\boldsymbol{\delta}$ is dry natural mode. If 1D analysis is applied the beam vibration modes should be spread to the ship wetted surface. The general expression for spreading nodal displacements to the wetted surface[10,11,12] yields:

$$\mathbf{h}=\left[\frac{\mathrm{d}w_v}{\mathrm{d}x}\left(Z-z_N\right)+\frac{\mathrm{d}w_h}{\mathrm{d}x}Y+\overline{u}\frac{\mathrm{d}\psi}{\mathrm{d}x}\right]\mathbf{i}+\left[-w_h-\psi\left(Z-z_S\right)\right]\mathbf{j}+\left[-w_v+\psi Y\right]\mathbf{k}. \tag{10}$$

where w_v is hull vertical deflection, w_h is hull horizontal deflection, ψ is twist angle, y and z are coordinates of the point on ship surface, and z_N and z_S are coordinates of centroid and shear centre respectively, and $\overline{u}=\overline{u}(x,y,z)$ is the cross-section warping intensity related to the wetted surface. Z and Y, represent global coordinates, while $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors, respectively. The expressions for nodal displacements of beam model and 3D FEM substructure model can be extracted from Eq. (11)

$$\delta_X=\frac{\mathrm{d}w_v}{\mathrm{d}x}\left(Z-z_N\right)+\frac{\mathrm{d}w_h}{\mathrm{d}x}Y+\overline{u}\frac{\mathrm{d}\psi}{\mathrm{d}x},\ \delta_Y=-w_h-\psi\left(Z-z_S\right),\ \delta_Z=-w_v+\psi Y. \tag{11}$$

These displacements are then imposed to the aft and fore 3D FEM substructure boundaries, and stress concentrations, as result of their differences, is calculated.

2.2 Hydrodynamic model

The full detail description of the used hydrodynamic model is given in a number of references[9,10,11], so only the brief description is given here. The used hydrodynamic model is solved in the frequency domain using the modal superposition method based on the potential theory assumptions. In that way hydrodynamic mass and damping matrices are extended to elastic modes and are defined as

$$A_{ij}=\rho\,\mathrm{Re}\iint_S\varphi_{Rj}\mathbf{h}_i\mathbf{n}\mathrm{d}S,\ B_{ij}=\rho\,\mathrm{Im}\iint_S\varphi_{Rj}\mathbf{h}_i\mathbf{n}\mathrm{d}S\,. \tag{12}$$

where ρ represents fluid density, φ_{Rj} is the radiation potential and $\mathbf{h}_i$ and $\mathbf{n}$ are dry natural mode and normal vector of the wetted surface S.

2.3 Hydrostatic model

Hydrostatic model takes into account the hydrostatic part of the total pressure obtained by Bernoulli's equation. There are several restoring stiffness formulations as for example formulation of Price & Wu[14], Newman's formulation[15], formulations of Huang & Riggs[16], Malenica[17], Senjanović et al[18,19]. Physically consistent formulation of restoring stiffness for ship structures, developed recently by Senjanović et al.[18], is used here.

Restoring stiffness represent the relation between excitation forces and displacements and is derived based on the modal superposition method using the variational or perturbation method. It consists of hydrostatic and gravity parts. The hydrostatic component is comprised of pressure and normal vector and mode parts that are defined as

$$C_{ij}^{p} = \rho g \iint_S \mathbf{h}_i h_z^j \mathbf{n} \mathrm{d}S,\; C_{ij}^{nh} = \rho g \iint_S Z\mathbf{h}_i \left(\nabla \mathbf{h}_j\right) \mathbf{n} \mathrm{d}S \,. \tag{13}$$

The gravity part of the restoring stiffness reads

$$C_{ij}^{m} = g \iiint_V \rho_s \left(\mathbf{h}_i \nabla\right) h_z^j \mathrm{d}V \,, \tag{14}$$

where ∇ is Hamilton differential operator. Finally, the complete restoring stiffness coefficients are obtained by summing up its constitutive parts

$$C_{ij} = C_{ij}^{p} + C_{ij}^{nh} + C_{ij}^{m} \,. \tag{15}$$

2.4 Hydroelastic model

The governing modal matrix differential equation for coupled ship motions and vibrations yields[10,11]:

$$\left[\mathbf{k} + \mathbf{C} - i\omega\left(\mathbf{d} + \mathbf{B}(\omega)\right) - \omega^2\left(\mathbf{m} + \mathbf{A}(\omega)\right)\right]\boldsymbol{\xi} = \mathbf{F} \,, \tag{16}$$

where **k**, **d**, and **m** are structural stiffness, damping and mass matrices, respectively, **C** is restoring stiffness, $\mathbf{B}(\omega)$ is hydrodynamic damping, $\mathbf{A}(\omega)$ is added mass, $\boldsymbol{\xi}$ are modal amplitudes, **F** is wave excitation and ω is encounter frequency. All quantities, except ω and ξ, are related to the dry modes.

3. COMPUTER SOFTWARE

Based on the outlined theory computer programs have been developed and checked by correlation analysis of the simulation results and the measured ones for a flexible segmented barge[12,20]. Hydroelastic model based on the coupling of 1D FEM and 3D BEM model and its wave response is formulated and solved by program HYELACS[21], Figure 3. Beam model is formulated by program STIFF[22], based on the theory of thin-walled girders. It calculates cross-section area, moments of inertias, shear areas, torsional modulus, warping and shear inertia modulus for both closed and open cross section.

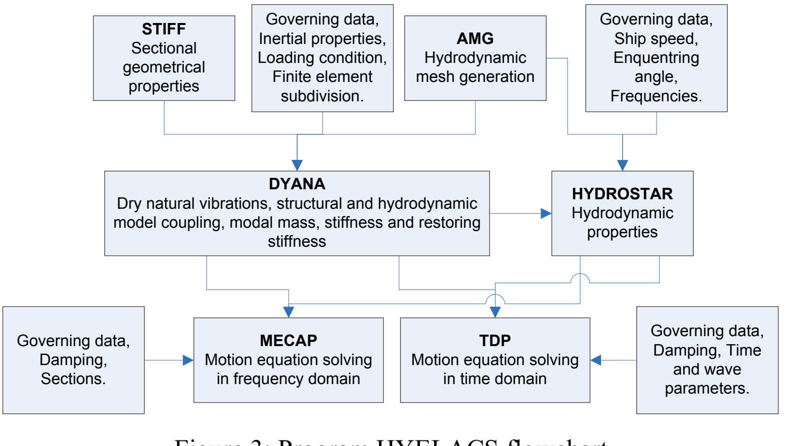

Figure 3: Program HYELACS flowchart

Model of the hydrodynamic wetted surface is automatically generated by program AMG[23]and is transferred to program DYANA[24] in order to calculate the modal restoring stiffness. Modal hydrodynamic properties are determined using program HYDROSTAR[25], and finally hydroelastic problem is solved by program MECAP[21]. Program HYELACS also allows solving problems of transient vibrations in time domain.

4. NUMERICAL EXAMPLE

4.1 Ship particulars

A large container ship of 11400 TEU is considered[4], Figure 4. The vessel length between perpendiculars, breadth and draft are 348 m, 45.6 m and 15.5 m, respectively.

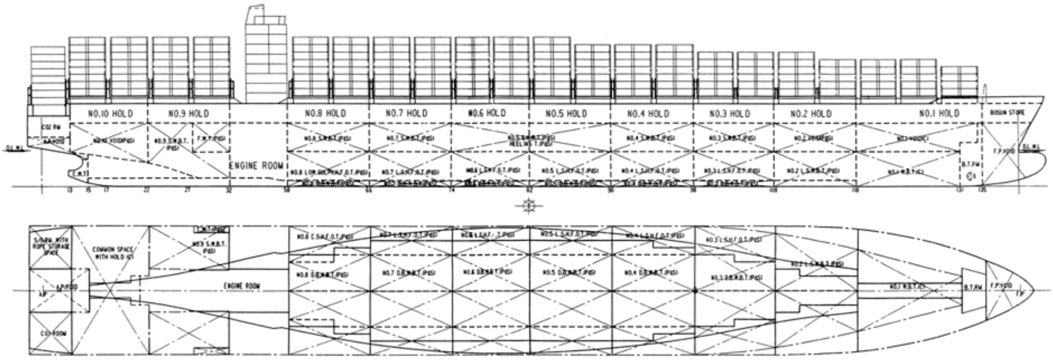

Figure 4: 11400 TEU container ship

4.2 Beam model verification

The reliability of the beam model is checked by comparing the lightship and full load dry natural frequencies, Table 1, and mode shapes, Figures 5 and 6 with the one obtained from 3D FEM analysis performed by NASTRAN[26]. Very good agreement of the results is found.

TABLE 1

DRY NATURAL FREQUENCIES OF THE LIGHT CONTAINER SHIP, w_i (HZ)

	1D		3D		Discrepancy, %	
No.	Vertical	Coupled	Vertical	Coupled	Vertical	Coupled
1	1.149	0.640	1.159	0.639	-0.86	0.16
2	2.318	1.053	2.328	1.076	-0.43	-2.14
3	3.694	1.738	3.654	1.750	1.09	-0,69

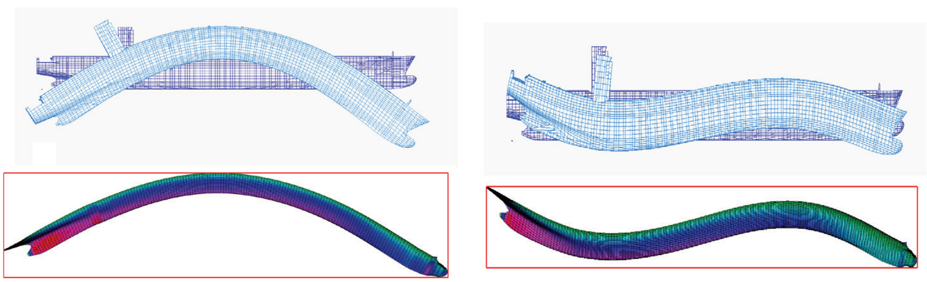

Figure 5: The 1st and the 2nd lightship vertical vibration natural mode, 3D and 1D

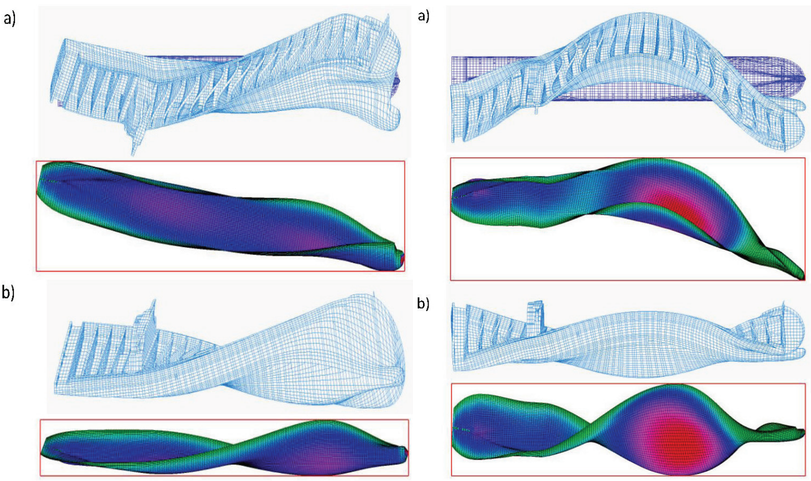

Figure 6: The 1st and the 2nd lightship coupled horizontal and torsional vibration natural mode, a) bird's eye, b) lateral view, 3D and 1D

4.3 Global hydroelastic response

Numerical calculation of ship response to waves is performed for several loading conditions, unit harmonic wave amplitude, and set of heading angles, ship speeds and wave lengths. Transfer functions of vertical bending moment, horizontal bending moment, and torsional moment at the amidships section, obtained using 1D FEM + 3D BEM hydroelastic model for the case of fully loaded ship, are shown in Figure 7. The angle of 180° corresponds to the head sea. They are compared to the rigid body ones determined by program HYDROSTAR[25]. Very good agreement is obtained in the lower frequency domain, where the ship behaves as a rigid body, while large discrepancies occur at the resonances of the elastic modes, as expected. Further on, hydroelastic response obtained by 1D FEM + 3D BEM hydroelastic model is compared to the one obtained by

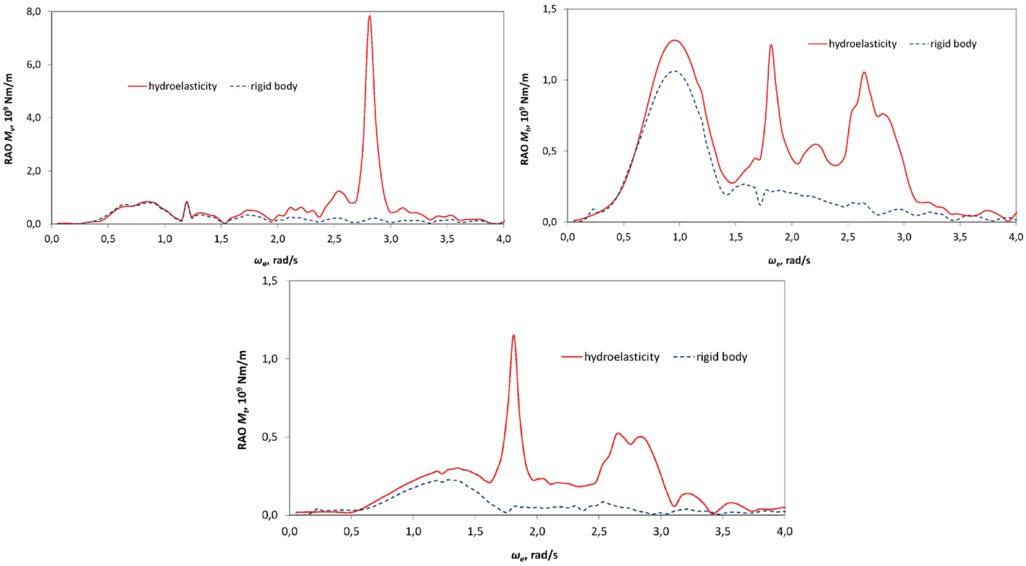

Figure 7: Transfer functions for vertical and horizontal bending moment and torsional moment, χ=120°, U=24.7 kn, x=175 m

fully coupled 3D FEM + 3D BEM hydroelastic model, Figure 8, where quite good agreement is obvious. In this particular case, the loading condition No. 7 from the trim and stability book and slightly lower ship speed was selected.

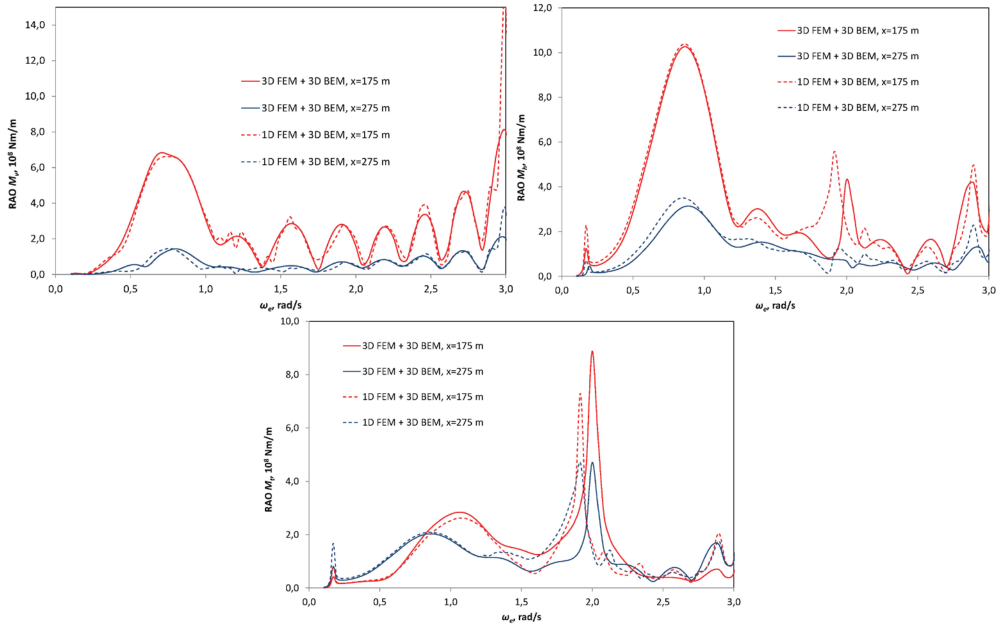

Figure 8: Transfer functions for vertical and horizontal bending moment and torsional moment, χ=120°, U=15.75 kn

4.4 Local response

The selected structural detail for stress concentration assessment is a knee in the hatch corner at the upper deck level in the middle part of the ship, Figure 9 and 10. For stress concentration analysis a 3D FEM substructure model with refined mesh in the vicinity of the selected structural detail, has been build in program NASTRAN[26]. It should be pointed out that the real and imaginary component of the response should be calculated separately, and at the end, at the level of stresses should be summed up as a complex quantity. Figure 10 shows the stress distributions in the considered structural detail for the selected frequency (ω = 0.9 rad/s). The analyzed stress is normal stress along the knee boundary. In order to register it, bar elements are fitted on the knee boundary. Transfer functions of stress concentrations obtained by 1D FEM +3D BEM and 3D FEM + 3D BEM hydroelastic models are presented in Figure 11. In the low frequency domain rather high discrepancies can be noticed, while in the high frequency domain, where the springing influence on fatigue damage accumulation is pronounced, quite good agreement is achieved.

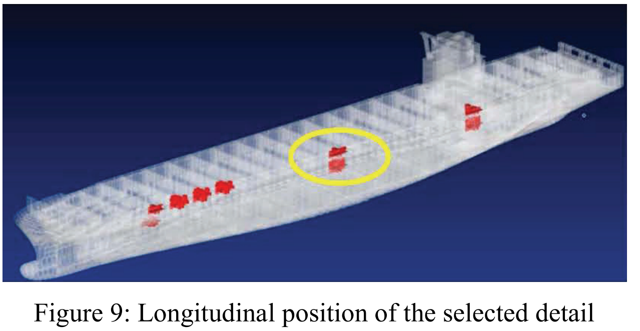

Figure 9: Longitudinal position of the selected detail

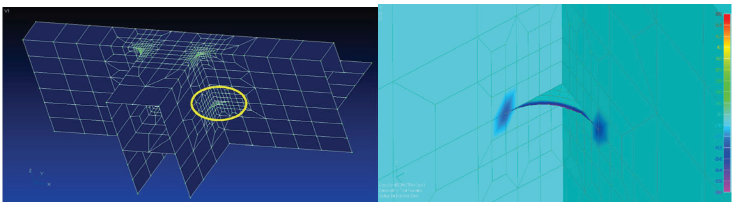

Figure 10: Selected structural detail and stress distribution

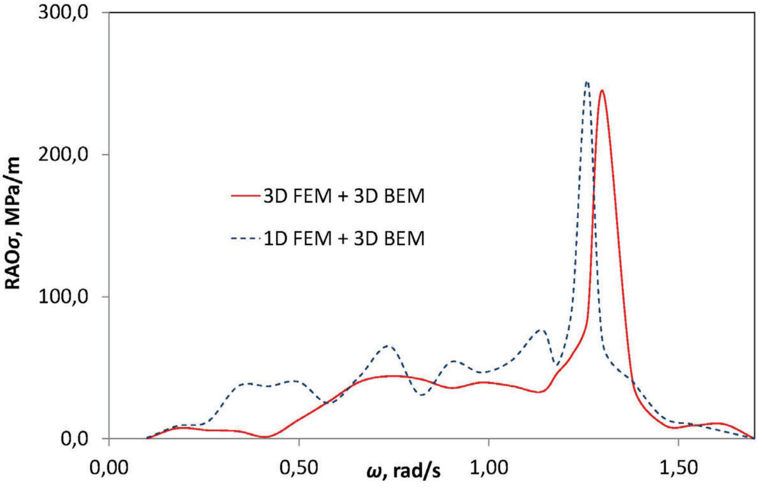

Figure 11: Transfer functions of stress concentrations

5. CONCLUSION

Beam structural models, commonly used in the preliminary design phase, represent powerful and reliable design tool. Therefore it is important to know the threshold of the beam model applicability. Due to that the existing methodology of ship hydroelasticity analysis was extended to assessment of fatigue damage of ship structural details. The improved methodology of ship hydroelasticity analysis was illustrated by one example, i.e. 11400 TEU container ship and after the verification of beam model importance of hydroelasticity analysis was demonstrated by comparison of the bending moment transfer functions for the case of rigid and flexible ship structure. Also, applicability of the beam structural model within the hydroelasticity methodology was demonstrated by comparison of the transfer functions of stress concentrations for the selected structural detail in the case of 3D and 1D FEM structural model. Although, very good agreement is achieved, especially in the high frequency range where springing influence is pronounced, some minor improvements in the low frequency domain could be done to increase the accuracy of fatigue damage calculation.

In the future work it is necessary to proceed further to ship motion calculation in irregular waves for different sea states, based on the known transfer functions. Also, model tests and full-scale measurements should be performed to enable the complete validation of the improved model and to extend the Classification Rules for the design and construction of ultra large container ships.

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